**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

**Answer: Time taken to service a car is 50 minutes.**

Pr(X > 50) = 1 - Pr(X ≤ 50).

Z = (X - 45)/8.0

Thus the question can be answered by using the normal table to find

Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**Answer – False**

If more employees are older than 44, this will shift the ***μ*** towards 44 with considering **sd**. Which is not possible as ***μ***is given 38 with sd 6.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Answer - True**

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Answer:**

As both are independent normal random variables,

X1 + X2 is normal with N(µ1+µ2,σ12+σ22).

And 2X1 will just scale the normal distribution by 2 times.

1. Let X ~ N(100, 202). Find two values, *a* and*b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. **48.5, 151.5**
6. 90.1, 109.9

**Answer –**

x=σ[z] + µ

So "a" = 0.5th percentile for X = 20\*[-2.57] + 100 = **48.5**

and "b" = 99.5th percentile for X = 20\*[+2.57] + 100 = **151.4**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Answer -**

>qnorm(0.025,45\*5,3)

[1] 219.1201

>qnorm(0.975,45\*5,3)

[1] 230.8799

>qnorm(0.025,45\*7,3)

[1] 309.1201

>qnorm(0.975,45\*7,3)

[1] 320.8799

So the Rupee range with 95% probability for the annual profit of the company is given by,

=Profit1 + Profit2

= [219.12, 230.87] + [309.12, 320.87]

= [528.24, 551.74]

So Profit of the company in Rupees(in **Million**) is between range **[528.24, 551.74]**

1. Specify the 5th percentile of profit (in Rupees) for the company

**Answer -**

>qnorm(0.05,45\*7,3)

[1] 310.0654

>qnorm(0.05,45\*5,3)

[1] 220.0654

5th percentile of profit = 310.0654+ 220.0654=**530.1308 Million Rs**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**Answer :** Division 2 with distribution N(7, 42)